A New Empirical Method for Extracting Unloaded Resonant Frequencies from Microwave Resonant Cavities

Antoni J. Canós, José M. Catalá-Civera, Felipe L. Peñaranda-Foix, Juan Monzó-Cabrera, and E. De los Reyes.

ITACA-Technical University of Valencia, E-46022, Spain

***Abstract* — Equivalent circuits traditionally used to model resonators and coupling networks in the vicinity of a resonance provide values of unloaded resonant frequencies which usually do not agree with experimental results. A new empirical method for the extraction of influence of coupling networks on resonant cavities is presented. The characterization of coupling structures is performed directly from measurements without the need of obtaining the electromagnetic fields inside the cavity, which is very interesting from a practical point of view. Results are validated with simulations and experimental measurements. The accuracy of some cavity applications, such as dielectric characterization techniques can be directly improved with this approach.**

I. Introduction

Precise characterization of resonant cavities (determination of unloaded parameters Q_u and f_u) is an essential task for many applications (filters, characterization of dielectric materials, etc.). Measuring a resonator involves using feeding mechanisms that modify its original (*unloaded*) response: resonance frequency is changed and quality factor is lowered. Therefore it is necessary to extract the effect of coupling networks from measurements [1].

Equivalent circuits traditionally used to model resonators and coupling networks in the vicinity of a resonance provide different procedures to extract unloaded resonant frequencies and Q_u of resonators, but these values usually do not agree with experimental results. Electromagnetic analyses used to evaluate accurately the influence of coupling network on resonant frequency [2], [3] also reveal resonant frequency deviations of opposite sign than those predicted by traditional circuital models.

In this work, a new empirical method for the extraction of influence of coupling networks on resonant cavities is presented. The characterization of coupling structures is performed directly from measurements without the need of obtaining the electromagnetic fields inside the cavity, which is very interesting from a practical point of view. A new element is added to the traditional equivalent circuit in order to model the real behavior of coupled resonators. Results are validated with simulations and experimental measurements. As a result, the accuracy of some cavity applications, such as dielectric characterization techniques can be directly improved with this approach.

II. Modeling Of Coupling Networks

Equivalent circuit in fig. 1a has been traditionally used to model one-port resonators coupled by lossless electrical feeding networks in the vicinity of a resonance and near the detuned open point [4], [5]. The influence of coupling network on resonance is represented by a parallel subsceptance b_e.

By using the representation of Fig. 1a, the relation between resonant frequencies with feeding mechanism, f_l , and without it, f_s , can be expressed as

$$f_l = \left(1 + \frac{b_e}{2 \cdot Q_e}\right) \cdot f_s \qquad (1)$$

where Q_e is Q-factor associated to the external elements of the structure [1].

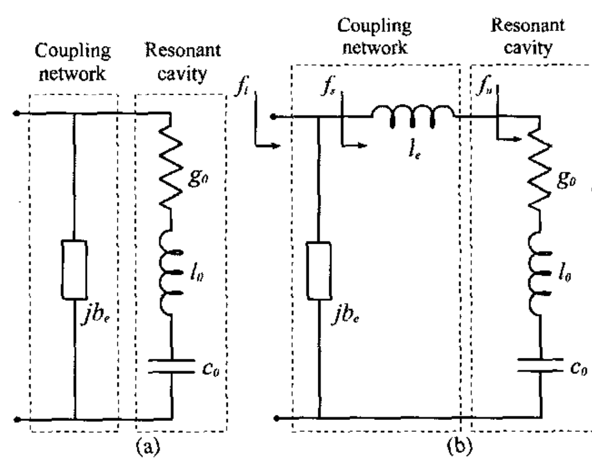

Fig. 1. Equivalent circuits in the vicinity of a resonance near the detuned-open point. a) Traditional representation. b) Suggested new representation.

The circuital model in Fig. 1a allows the obtaining of unloaded Q-factors of resonators from measurements but,

0-7803-7695-1/03/$17.00 © 2003 IEEE

as mentioned, predicted values of resonant frequencies don't agree sometimes with experiments, as shown bellow. In consequence, unloaded frequencies can not be successfully obtained from measurements. It is experimentally observed that b_e takes positive values (decoupled reflection coefficient defined in [4], is clockwise shifted from open-point). Increasing the perturbation introduced by coupling networks on ideally isolated resonators (e.g. using longer probes) increases the value of b_e and decreases Q_e. According to (1), resonance frequency should be increased ($f_l > f_s$) by the effect of coupling network. It is stated from measurements that the stronger is the perturbation (longer probe), the lower is the measured frequency, f_l , contrariously as expected from previous model, so f_s in (1) is not the resonant frequency of the isolated cavity. For extremely undercoupled resonators ($Q_e \rightarrow \infty$) resonant frequencies are higher and tend to theoretical values.

Fig. 1b, shows the suggested new equivalent circuit, where a serial inductor l_e is added to model the lowering of frequency due to the coupling network.

Responses of resonant cavities, coupled by electrical probes of different length were measured. The effects of coupling mechanism on resonant frequencies were modeled with circuit of fig 1a and extracted by means of Kajfez procedure [1], [6]. As shown bellow, obtained values of frequency (f_s) still depend on coupling factor and don't agree with theoretical expected values calculated from physical dimensions of the cavity. Since the influence of feeding mechanisms is not completely extracted, resonant frequencies obtained with previous models are called in this paper as *semi-loaded* frequencies, f_s.

Therefore equivalent circuit of fig. 1a represents reactive effects of coupling networks, but it may not model other perturbations on cavities as apertures or invasions of the cavity with probes. *Loaded*, *semi-loaded* and *unloaded* resonant frequencies (each of them affected for different effects of coupling network) are indicated in fig. 1b.

It was empirically determined that values of f_s follow the expression:

$$f_s = \left(1 - \frac{A}{Q_e^{\alpha}}\right) \cdot f_u \qquad (2)$$

In this expression, frequency f_u is the value towards f_s tends when $Q_e \rightarrow \infty$. Values of f_u agree with theoretical expressions from cavity dimensions, so they are referred as *unloaded* resonant frequencies, since they are characteristics of ideally isolated resonators and the influence of coupling network has been totally extracted. Parameters A and α depend on electromagnetic fields into the cavity (i.e. cavity dimensions, resonant mode, type and location of coupling mechanisms) and they can be directly determined from measurements.

Three or more coupling networks of the same type and different coupling factor are needed for the determination of f_u , A and α . Values of Q_e and f_s, can be obtained by Kajfez procedure [1], [6]. Better results are obtained with networks producing dissimilar coupling factors (e.g. undercoupled, critically coupled and overcoupled configurations).

III. RESULTS AND VALIDATION

To validate the method described above, the responses of a rectangular cavity (100.1 x 20 x 340.4 mm), coupled by several electrical probes from a coaxial cable were measured for TE_{103} mode. The cavity was characterized using three probes (circles in Fig. 2), obtaining the values A = 0.6556, α = 0.7386 and f_u = 1.996820 GHz (dashed line in Fig. 2), which agrees with theoretical value obtained from physical dimensions of the closed resonator (1.996897 GHz). Another probes feeding the cavity yield f_s values (crosses in Fig. 2) in agreement with expression (2) (solid line in Fig. 2). After characterized, accurate unloaded resonant frequency of the cavity can be extracted from any coupling probe, as shown in Table I.

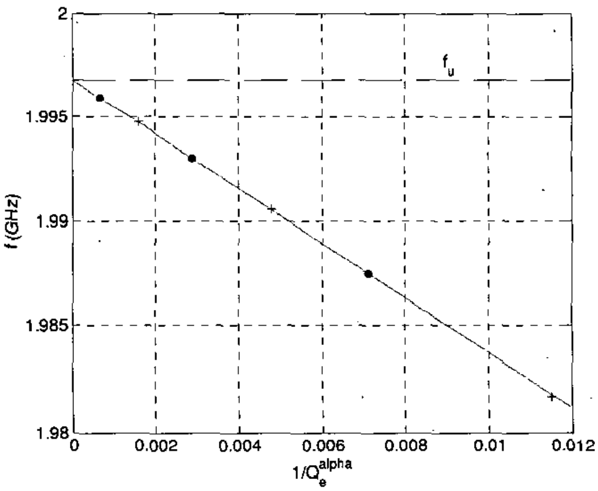

Fig. 2. Resonant frequencies vs external Q-factor. Semi-loaded frequencies for coupling network characterization in circles. Semi-loaded frequencies for the rest of probes in crosses. Predicted values in a solid line. Unloaded frequency in dashed line.

A rectangular partially-filled cavity coupled by electrical probes of different length was simulated with the FDTD electromagnetic simulator CONCERTO® [7]. The

TABLE I
MEASURED AND EXTRACTED VALUES OF RESONANT FREQUENCIES OF AN EMPTY CAVITY

Probe	f_l (GHz)	b_e	Q_e	f_s (GHz) [6]	f_u (GHz)
1	1.995966	0.048	20508	1.995964	1.996820
3	1.993092	0.107	2757.5	1.993053	1.996820
5	1.987707	0.155	810.61	1.987517	1.996820
2	1.994717	0.083	6122.4	1.994703	1.996793
4	1.990647	0.128	1381.9	1.990555	1.996828
6	1.982054	0.187	421.31	1.981614	1.996697

TABLE II
SIMULATED AND EXTRACTED VALUES OF RESONANT FREQUENCIES OF A PARTIALLY-FILLED CAVITY

Probe length	f_l (GHz)	b_e	Q_e	f_s (GHz) [6]	f_u (GHz)
1 mm	1.964576	0.085	89029	1.964575	1.964690
4 mm	1.963778	0.173	7866.9	1.963757	1.964633
8 mm	1.960778	0.249	1263.5	1.960584	1.964620
10 mm	1.957907	0.361	625.59	1.957342	1.964646

resonant frequency of a simulation with a closed cavity excited by a lumped element is taken as a reference value, since the cavity remains isolated. The cavity was characterized using three probes, obtaining the values A = 0.8129, $\propto$ = 0.8370 and f_u = 1.964633 GHz, which agrees with the reference value (1.964618 GHz). After characterized, accurate unloaded frequency of the cavity can be extracted from any coupling probe, as shown in Table II.

IV. CONCLUSIONS

Equivalent circuits traditionally used to model resonators and coupling networks in the vicinity of a resonance provide values of unloaded resonant frequencies that usually do not agree with experimental results. A new empirical method for the extraction of influence of coupling networks on resonant cavities is presented. The characterization of coupling structures is performed directly from measurements without the need of obtaining the electromagnetic fields inside the cavity, which is very interesting from a practical point of view. A new element is added to the traditional equivalent circuit in order to model the real behavior of coupled resonators. Results were validated with simulations and experimental measurements. As a result, the accuracy of some cavity applications such as dielectric characterization techniques can be directly improved with this approach.

REFERENCES

[1] D. Kajfez, *Q Factor*, Oxford, MS: Vector Fields, 1994.
[2] T. Koryu Ishii, *Handbook of Microwave Technology. Vol I: Components and Devices*, San Diego: Academic Press, 1995. Ch 3 (by R.A. Strangeway).
[3] R.E. Collin, *Foundations for Microwave Engineering*, New York: McGraw-Hill, 1992.
[4] D. Kajfez and E.J. Hwan, "Q-Factor measurement with network analyzer", *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-32, no.7, pp. 666-670, July 1984.
[5] E.-Y. Sun and S.-H. Chao, "Unloaded Q measurement – The Critical-Points Method", *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-43, no.8, pp. 1983-1986, August 1995.
[6] D. Kajfez, "Linear fractional curve fitting for measurement of high Q-factors", *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-42, no.7, pp. 1149-1153, July 1994.
[7] *Concerto User Guide*, Vector Fields Limited, Oxford, England, 1999.